NAG Toolbox for MATLAB

s14ba

1 Purpose

s14ba computes values for the incomplete gamma functions $P(a,x)$ and $Q(a,x)$.

2 Syntax

```
[p, q, ifail] = s14ba(a, x, tol)
```

3 Description

s14ba evaluates the incomplete gamma functions in the normalized form

$$P(a,x) = \frac{1}{\Gamma(a)} \int_0^x t^{a-1} e^{-t}\, dt,$$

$$Q(a,x) = \frac{1}{\Gamma(a)} \int_x^\infty t^{a-1} e^{-t}\, dt,$$

with $x \geq 0$ and $a > 0$, to a user-specified accuracy. With this normalization, $P(a,x) + Q(a,x) = 1$.

Several methods are used to evaluate the functions depending on the arguments a and x, the methods including Taylor expansion for $P(a,x)$, Legendre's continued fraction for $Q(a,x)$, and power series for $Q(a,x)$. When both a and x are large, and $a \simeq x$, the uniform asymptotic expansion of Temme 1987 is employed for greater efficiency – specifically, this expansion is used when $a \geq 20$ and $0.7a \leq x \leq 1.4a$.

Once either P or Q is computed, the other is obtained by subtraction from 1. In order to avoid loss of relative precision in this subtraction, the smaller of P and Q is computed first.

This function is derived from the (sub)program GAM in Gautschi 1979.

4 References

Gautschi W 1979 A computational procedure for incomplete gamma functions *ACM Trans. Math. Software* **5** 466–481

Gautschi W 1979 Algorithm 542: Incomplete gamma functions *ACM Trans. Math. Software* **5** 482–489

Temme N M 1987 On the computation of the incomplete gamma functions for large values of the parameters *Algorithms for Approximation* (ed J C Mason and M G Cox) Oxford University Press

5 Parameters

5.1 Compulsory Input Parameters

1: **a – double scalar**

The argument a of the functions.

Constraint: $\mathbf{a} > 0.0$.

2: **x – double scalar**

The argument x of the functions.

Constraint: $\mathbf{x} \geq 0.0$.

3: **tol – double scalar**

The relative accuracy required by you in the results. If s14ba is entered with **tol** greater than 1.0 or less than ***machine precision***, then the value of ***machine precision*** is used instead.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **p – double scalar**
2: **q – double scalar**

The values of the functions $P(a,x)$ and $Q(a,x)$ respectively.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **a** ≤ 0.0.

ifail $= 2$

On entry, **x** < 0.0.

ifail $= 3$

Convergence of the Taylor series or Legendre continued fraction fails within 600 iterations. This error is extremely unlikely to occur; if it does, contact NAG.

7 Accuracy

There are rare occasions when the relative accuracy attained is somewhat less than that specified by parameter **tol**. However, the error should never exceed more than one or two decimal places. Note also that there is a limit of 18 decimal places on the achievable accuracy, because constants in the function are given to this precision.

8 Further Comments

The time taken for a call of s14ba depends on the precision requested through **tol**, and also varies slightly with the input arguments a and x.

9 Example

```
a = 2;
x = 3;
tol = 1.111307226797642e-16;
[p, q, ifail] = s14ba(a, x, tol)
```

```
p =
    0.8009
q =
    0.1991
ifail =
           0
```